STRONG GRAVITATIONAL LENSING AND THE STRUCTURE OF QUASAR OUTFLOWS

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ABSTRACT

We show that by analyzing the spectra of lensed broad absorption line quasars (BALQSOs), it is possible to reveal key properties of the outflowing gas in the inner regions of these objects. This results from the fact that each image of the quasar corresponds to a different line of sight through the outflow. This combined with dynamical estimates for the location of the flow, adds new information concerning the lateral, non line of sight structure of the absorbing gas. Here we consider a sample of lensed BALQSOs and note that the similarity of BAL profiles of different images of the same quasar implies that the flow is relatively isotropic on small scales. We show that its geometry is inconsistent with the ballistically accelerated spherical cloud model, and that wind models provide a better description of the flow structure. Furthermore, observations seem to disagree with naive interpretations of recent time-dependent wind simulations. This may hint on several important physical processes that govern the structure and dynamics of such flows. Future prospects for the study of quasar outflows with the effect of strong gravitational lensing are discussed.

Subject headings: gravitational lensing — ISM: jets and outflows — galaxies: active — galaxies: nuclei — quasars: absorption lines

1. INTRODUCTION

The standard model for quasars consists of a central continuum emitting region (possibly an accretion disk), broad and narrow line regions (BLR and NLR respectively), and a torus. Among these components only the NLR can be resolved in nearby quasars to reveal a stratified structure and a cone-shape geometry. The physical nature of the other components is subject to an ongoing debate, e.g., it is not known whether the BLR is in the form of virialized clouds (e.g., Kaspi & Netzer 1999) or a continuously outflowing wind (e.g., Murray, Chiang, Grossman, & Voit 1995; hereafter MCGV).

Observations indicate that outflowing gas is a common phenomenon in quasars. Such gas is intrinsic to the quasar, as has been shown for the high velocity ($\gtrsim 10^3 \ \text{km s}^{-1}$), broad absorption line (BAL) flows in BALQSOs (Weymann 1977). The quasar unification scheme (e.g., Elvis 2000) suggests that BAL flows are ubiquitous and their detection (in some 15% of type-I objects; Hewett & Foltz 2003) requires a fortuitous line of sight to the central engine.

There have been several unsuccessful attempts to constrain the location of BAL flows via the detection of variations in velocity or ionization. de Kool et al. (2001) have detected absorption from excited levels of iron which allowed location estimates in a subclass of BALQSOs (FeLoBALQSOs). An indirect measure of the gas location relies on the observationally motivated assumption that it is accelerated by radiation pressure force (Arav 1996). This has been the focus of many works concerning the dynamical modeling of such flows.

Dynamical models consist of cloud (e.g., Chelouche & Netzer 2001, hereafter CN01), wind (MCGV, Proga, Stone, & Kallman 2000; hereafter PSK), and cloud-wind models (e.g., Arav, Li, & Begelman 1994; hereafter ALB). Some versions of such models can be brought to some agreement with observations. They do not however agree among themselves concerning the mass flow rate and other important parameters. Thus, despite some 30 years of research, several key proper-

ties of BAL flows remain unknown.

In this *Letter* we show that strong lensing can add invaluable information on the structure of BAL flows: In §2 we define our sample of lensed BALQSOs. §3 discusses the observational constraints on several flow models. Future prospects and caveats are discussed in §4. Summary follows in §5.

2. LENSED BALOSOS: GEOMETRY AND SAMPLE

In gravitationally lensed systems, each image corresponds to a different line of sight to the central object. For typical lens configurations (figure 1), the angle between the line of sights, $\Delta \varphi$, is of order of the separation angle between the images ($\Delta \theta$; see e.g., Schneider, Ehlers, & Falco 2000).

We consider 7 lensed and one lens candidate (UM 425; see however Aldcroft & Green 2003 who favour a binary quasar interpretation for this system) BALQSOs whose properties were taken from CASTLES² (Muñoz et al. 1998) and Michalitsianos et al. (1997), and are given in table 1. Assuming a typical type-I quasar continuum with $\alpha_{ox}=1.4$ (Green et al. 1995) and a X-ray (0.1-10keV) continuum slope, $\alpha_x=0.9$, the mean demagnified bolometric luminosity of the quasars in our sample is $\sim 10^{46}$ erg s⁻¹ (Chartas 2000; this value depends on the assumed lens model and is somewhat insecure for double imaged lenses).

Spectroscopic observations show that in most lensed BALQSOs, the absorption troughs are similar between different images to within S/N and resolution limits. This is probably not a selection effect (due to the search criteria used for finding lensed objects) since the fraction of BALQSOs among lensed quasars is larger (Chartas 2000; or similar but not smaller) than their fraction among quasars (Hewett & Foltz 2003). This means that BAL flows are relatively isotropic on small angular scales. Nevertheless, for two objects in our sample (APM 08279+5255 and the lens candidate UM 425) there seem to be noticeable differences in the BAL troughs between the images. We discuss the implications of these results on various dynamical models in §3.

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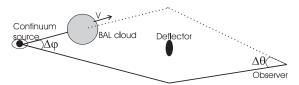


FIG. 1.— A schematic (not to scale) view of a double image lens where each line of sight to the continuum source traverses different sections of the BAL flow (see text).

3. IMPLICATIONS FOR QUASAR OUTFLOW MODELS

Here we show that by comparing BAL features of different images one can put constraints on existing flow models. This results from the different characteristic angular scale of the flow predicted by different models. Below we discuss each class of models separately.

3.1. Single cloud models

This model was initially invoked to explain the BLR gas and only later applied to BAL flows. Several works (e.g., Blumenthal & Mathews 1979, hereafter BM79) have demonstrated a qualitative agreement between model and observations in terms of the flow velocity and spectral features.

In these models the clouds are spherical to a good approximation (BM79) and are confined by some external pressure, $P(P \propto r^{-2})$ is typically assumed where r is the distance from the central object; ALB). The clouds are photoionized by the quasar radiation field and their ionization structure depends primarily on the ionization parameter, U_x (defined as the ratio of photon density in the range 0.1-10 keV to gas number density; CN01). The terminal velocity of the cloud (v_{10000} ; in units of 10^4 km s⁻¹) is given by

$$v_{10000} \simeq 0.1 L_{45}^{1/2} r_{17}^{-1/2} M^{1/2},$$
 (1)

where $r_{17} = r/10^{17}$ cm is the launching distance, L_{45} the bolometric luminosity in units of 10^{45} erg s⁻¹, and M the "force multiplier" (the ratio of the total to the Compton radiation pressure force; M(r) = const. for the assumed pressure profile; CN01). For $10^{-5} < U_x < 0.1$ and the typical velocities and column densities of BAL flows, M can be approximated up to a factor ~ 3 by

$$M \simeq 0.3ZU_{\rm r}^{-1},\tag{2}$$

where *Z* is the gas metalicity normalized to solar values.

High S/N spectroscopy combined with detailed photoionization calculations can constrain the ionization parameter, U_x and the column density N_{21} (in units of 10^{21} cm⁻²; assuming Z=1) of the outflowing gas. These seem to cluster around $U_x \sim 10^{-4}$, $N_{21} \gtrsim 1$ for BAL flows (Arav et al. 2001) and $U_x \sim 10^{-5}$, $N_{21} \gtrsim 1$ for FeLoBAL flows (de Kool et al. 2002). Therefore, high ionization BAL flows that are launched from the centers ($r_{17} \gtrsim 50$) of luminous quasars ($L_{45} = 10$) would reach a terminal velocity $\lesssim 15,000$ km s⁻¹ (equations 1 & 2) which is in qualitative agreement with observations.

Combining equations 1 & 2 we obtain,

$$r_{17} \simeq 3 \times 10^{-3} v_{10000}^{-2} L_{45} Z U_x^{-1},$$
 (3)

where v_{10000} is inferred from the blue wing of BAL troughs. This estimate for r_{17} can be compared to those derived for FeLoBALQSOs: For FBQS 1044+3656 ($L_{45}=10~v_{10000}\simeq 0.5$) de Kool et al. (2002) conclude that $r_{17}\simeq 2\times 10^4$ (see however Everett, Königl, & Arav 2002) which is consistent with the dynamical estimate for this object ($r_{17}\sim 10^4$).

TABLE 1
PROPERTIES OF THE LENSED BALOSOS SAMPLE

Object	Z_S	z_l	$\Delta \theta$	t_{lag}	v ₁₀₀₀₀	Refs.
[1]	[2]	[3]	[4]	[5]	[6]	[7]
APM 08279+5255	3.91	-	0.38"	-	1	a,b
SBS 1520+530	1.86	0.72	1.56"	130	2	c,d
HE 2149-2745	2.03	0.49	1.7"	103	0.8	c,e
RX J 0911.4+0551	2.8	0.77	0.48"	146	0.3	c,e
PG 1115+080	1.72	0.37	2.3"	25	0.6	c,f
UM 425	1.46	0.6	6.5"	-	1.2	g,h,i
H 1413+117	2.55	_	1.35"	-	0.9	c,j
J 1004+1229	2.66	0.95	1.54"	-	1	k

The sample of lensed (and the lens candidate UM 425) BALQSOs: [1] Object's name, [2] Source redshift, [3] Lens redshift, [4] Image separation (arcseconds), [5] Time lag between images (days), [6] Maximum flow velocity (10000 km s $^{-1}$) inferred from the absorption line troughs, [7] References: (a) Chartas et al. 2002, (b) Irwin et al. (1998), (c) Chartas (2000), (d) Burud et al. (2002), (e) Bade et al. (1997), (f) Michalitsianos et al. 1996, (g) Michalitsianos et al. 1997, (h) Green et al. (2001), (i) Aldcroft & Green (2003), (j) Hutsemekers (1993), (k) Lacy et al. (2002)

It follows from the definition of U_x that the angle subtended by a spherical cloud with a column density $Z^{-1}N_{21}$ (e.g., Arav et al. 2001) is

$$\Delta \varphi_c \simeq 10 U_x r_{17} L_{45}^{-1} Z^{-1} N_{21}$$
 arcseconds. (4)

By combining equations 3 and 4 we obtain

$$\Delta \varphi_c \simeq 3 \times 10^{-2} v_{10000}^{-2} N_{21} \text{ arcseconds.}$$
 (5)

Thus, for all objects in our sample the cloud's lateral size, $\sim 10^{11} r_{17} \Delta \varphi_c$ cm, is smaller than the Schwartzschild radius (R_s) of the central black hole (BH; using the luminosity–BH mass relation of Kaspi et al. 2000). The fact that BAL flows only partially cover the ionizing source (whose size is $\sim 10R_s$ for a standard accretion disk; Arav et al. 2001) corroborates our size estimates.

Inspection of table 1 reveals that for all objects, $\Delta \varphi_c \ll \Delta \theta$. For clouds to show identical absorption features in all quasar images, their aspect (lateral to radial) ratio should reach $\sim 10^3$ for the more extreme cases in our sample (the cloud model is however consistent with the different BAL troughs in e.g., UM 425; Aldcroft & Green 2003). Such oblate clouds would probably be destroyed over dynamical timescales and a coherent acceleration is therefore unlikely (BM79).

Thus far, we have considered single clouds. Nevertheless, the flow column density may be distributed among many clouds (this is likely given the smooth nature of BAL troughs). For simplicity we assume that the flow consists of n identical clouds, each with a column $Z^{-1}N_{21}/n$. In this case, $\Delta \varphi_c$ is reduced by a factor n. This model is sometimes referred to as the tube model where the aspect ratio of the flow is much smaller than unity (contrary to what is implied by lensed quasar observations). The probability for a perfect alignment of the tube to several lines of sight is negligibly small . Thus, more likely geometries are those of a sheet or a cone which are naturally accounted for by wind models.

3.2. Clumpy wind models

In this model (ALB), the flow is assumed to consist of numerous small clouds ("cloudlets"). Isotropy is assumed in the statistical sense, i.e., each line of sight corresponds to a *different* realization of the *same* cloud distribution. Thus, each image of the quasar may show different absorption features due to statistical fluctuations in the number of clouds between different lines of sight which follow the Poissonian distribution. Following ALB, we assume identical cloudlets. Denot-

ing the total optical depth by τ and the fluctuations in the optical depth by $\Delta \tau$, one can estimate (assuming 1σ fluctuations) the average number of clouds along the lines of sight,

$$n \simeq \tau^2 \Delta \tau^{-2} \tag{6}$$

for $\Delta \tau < \tau$. As the spectral signature of a single cloud cannot extend considerably beyond one thermal width in velocity space (typically $\sim 10~{\rm km~s^{-1}}$) it allows us to constrain, or put lower limits to the number of clouds in each velocity bin for non-saturated troughs. In most cases, BAL profiles of different images are identical within S/N limits implying that the number of clouds in each velocity bin is $\gg 1$ and their total number $\gg 100$. This information is orthogonal to that concerning the filling factor of flows. Furthermore, observing systematic differences over a wide velocity range would indicate changes in the global flow properties (in the cloud distribution function). Such variations seem to be implied by the dissimilarities in BAL troughs for two objects in our sample (APM 08279+5255 and UM 425; see §4.1) although higher quality data are needed to confirm this point.

3.3. Continuous wind models

Time independent continuous wind models usually assume gas which expands according to the spherical continuity condition $\rho \propto \dot{\Omega} r^{-2} v^{-1}$ (ρ , v being the gas density and velocity respectively, and Ω the opening angle; e.g., MCGV). This assumption is justified for $\Omega \lesssim 4\pi$. However, flows are likely to evaporate quickly through their rims for considerably smaller Ω , since the expansion timescale ($\propto \sqrt{\Omega}r_{17}/v_s$, $v_s \simeq 20 \, \mathrm{km \ s^{-1}}$ is the sound speed) is smaller than the dynamical timescale ($\propto r_{17}/v_{10000}$). Put differently, for $\sqrt{\Omega/4\pi} \ll$ $400v_{10000}^{-1}$ arcseconds, the gas would evaporate before reaching its terminal velocity. Therefore, a confining pressure must be invoked, and it is highly unlikely that the resulting continuity condition would mimic a spherical expansion. If the differences in BAL troughs in APM 08279+5255 and UM 425 are real, these imply that $\sqrt{\Omega/4\pi} \simeq \Delta\theta$ (otherwise the probability for detecting such differences would be negligible). Thus, at least for these objects (see also Chelouche & Netzer 2003), the assumption of spherically expanding flows is unwarranted and alternative geometries with very different spectral predictions are more appropriate [e.g., confined (Königl & Kartje 1994) and freely expanding flows (Chelouche 2003)].

3.4. Time dependent wind models

Time-dependent calculations (PSK) show that Rayleigh-Taylor, Kelvin-Helmholtz, and radiative instabilities result in a filamentary sub-structure of the flow. Currently, there are no spectral predictions for their model and direct comparison with observations is complicated. Nevertheless, the results for the angular distribution of U_x and density (see figure 4a,b in PSK for the angular range subtended by high velocity gas) imply that the extreme (> 10 orders of magnitude!) variations over $\lesssim 10^{\circ}$ result from opacity rather than density effects. Thus, U_x is a measure of the transmitted spectrum. Assuming that U_x is a continuous function of angle down to small scales, then interpolation of PSK results predicts it should differ by many orders of magnitude over ~1 arcsecond scales. Such extreme variations would result in different spectral signatures between quasar images which are not observed. A more detailed study of this effect (e.g., by accounting for the finite size of the continuum source and the stratified flow structure) is

limited by the low resolution of current simulations. This apparent contradiction between model and observations may be resolved if there is a typical scale (larger than $r\Delta\theta \sim 10^{14}$ cm) below which variations are suppressed (e.g., due to the dissipation of turbulence below some scale or by the stablizing effect of magnetic fields; e.g., Proga 2003).

4. FUTURE PROSPECTS

We have demonstrated a new method for investigating the structure of BAL flows. Similar reasoning can be applied to additional types of flows namely, narrow absorption line (NAL) and X-ray BAL (XBAL) flows. For NAL flows $N_{21}\sim 2$ (e.g., Kraemer et al. 2002; but see also Hamann, Netzer, & Shields 2000 for lower values) and $v_{10000}\lesssim 0.1$ (Crenshaw et al. 1999) and, therefore, $\Delta \phi_c \gtrsim 2$ arcseconds, i.e., the single cloud model cannot be firmly ruled out for typical lens separations. This stems from the fact that such gas lies at larger distances and is more dilute. XBALs have so far been observed in two objects (APM 08279+5155 and PG 1115+080; Chartas et al. 2002, 2003) and seem to have $N_{21}=10^2$, and $v_{10000}=10$. This gives $\Delta \phi_c \sim 10^{-2}$ arcsecond. Nevertheless, due to the complications associated with X-ray spectroscopy, it is difficult to compare XBALs of different images.

Given several lines of sight, it may be possible to detect whether the flow is collimated and at what direction. From pure geometrical considerations, a collimated flow will show a velocity difference $\delta v \simeq v \delta \varphi$ between two lines of sight $\delta \varphi$ apart. For SBS 1520+530, $\delta v \sim 0.1~{\rm km~s^{-1}}$, i.e., within the resolution of next generation spectrometers. This, combined with an independent measure for the orientation of the quasar (e.g., via the detection of a jet), may allow us to ascertain whether BAL flows are predominantly polar or planar and how they relate to other components of the quasar.

Outflows are likely to rotate around the central object (e.g., MCGV) and it may be possible to detect spectral absorption features sweeping across our line of sight over Keplerian timescales,

$$\tau_{\text{Kep.}} \simeq 10^{-6} v_{10000}^{-3} L_{45} Z^{3/2} U_x^{-3/2} \Gamma^{1/2} \Delta \varphi \, \text{days},$$
(7)

(Γ is the luminosity in units of the Eddington luminosity). Typically $\tau_{\rm Kep.} \sim 100\,{\rm days}$ for BAL flows in quasars with $\Gamma=0.1$. Detecting such events would set more stringent limits on the geometry of the outflowing gas and may reveal the angular momentum balance in the inner quasar regions (perhaps even in the accretion disk). Such studies may also provide better estimates for H_0 since absorption events do not depend on the global geometry of the system and are less smeared in time compared to emission events.

4.1. Caveats

We have assumed that flow properties do not change on timescales shorter than the time lag, $t_{\rm lag}$, between quasar images. This requires that both the dynamical timescale, $t_{\rm dyn}$ and the intrinsic flux variability timescale, $t_{\rm flux}$, are longer than $t_{\rm lag}$. Indeed, the dynamical timescale, $t_{\rm dyn} \simeq v_{10000}^{-3} L_{45} Z U_x^{-1}$ days is much longer than $t_{\rm lag}$ (table 1) for UV flows ($t_{\rm dyn} \ll t_{\rm lag}$ for XBALs) and recent variability studies of high redshift quasars indicate that $t_{\rm flux} \gg t_{\rm lag}$ (Kaspi et al. 2003). Thus, any differences between UV BAL troughs are probably not related to time-lag effects.

An additional complication arises from micro-lensing (ML) by stars in the lensing galaxy which can differentially (de)magnify the continuum emitting region (e.g., Wambsganss

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1992). Its effect, being line of sight dependent (e.g., Ofek & Maoz 2003), may alter the shape of non-black troughs and result in apparent discrepancies between them. A more quantitative discussion requires further study (Chelouche 2003, in preparation) and is beyond the scope of this *Letter*.

5. CONCLUSIONS AND SUMMARY

In this *Letter* we show that by analyzing the spectral absorption features of spatially resolved lensed BALQSOs, one can obtain invaluable information concerning the geometry and sub-structure of BAL flows. Given the currently available spectroscopic data we conclude that such flows are relatively isotropic on a few arcsecond scales. Our analysis suggests that the coherently accelerated cloud model (and the related tube model) are inconsistent with observations, and that BAL flows are probably made of one or several outflowing sheets or cones with lateral dimensions much larger than the typical separation between quasar images. The similarity

of the absorption features between different images is inconsistent with naive interpretations of time-dependent dynamical simulations. This may indicate a typical length scale below which sub-structure is suppressed. Future high resolution spectroscopy of lensed BALQSOs may reveal whether such flows are collimated and distinguish between polar and planar flow configurations. Multi-epoch observations may put further constraints on the lateral geometry and dynamics of BAL flows as well as lead to better estimates for H_0 . Such information cannot be obtained from observations of non-lensed BALQSOs, and is imperative to our understanding of quasar structure and the physical processes near black-holes.

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